Short Papers

Analysis of Transient Interaction of Electromagnetic Pulse with an Air Layer in a Dielectric Medium Using Wavelet-Based Implicit TDIE Formulation

Yair Shifman and Yehuda Leviatan

***Abstract*—The interaction of transient electromagnetic pulse with an air layer in a dielectric medium is formulated in terms of a time-domain integral equation and solved numerically via the method of moments. Previous related works pointed to the inherent inadequacy of the marching-on-in-time method in this case, but suggested no remedy. This paper explains why an implicit modeling scheme would work effectively in this case. It is also noted that the use of an implicit scheme would normally involve a solution of a very large and dense matrix equation. To alleviate this drawback of the implicit scheme, the use of a wavelet-based impedance-matrix-compression technique, which has facilitated in the very recent past solutions of time-domain problems with greater efficiency, is described.**

***Index Terms*—Air layer, implicit formulation, time-domain integral equation, transient analysis, wavelet.**

I. Introduction

Analysis of transient interaction of an electromagnetic pulse with a layer, characterized by a dielectric constant lower than that of the surrounding medium, is often required, e.g., in the study of human tissues and the investigation of underground air tunnels and inner faults in structures [1], [2]. This interaction problem can be formulated in terms of a time-domain volume integral equation, using the Green's function of homogeneous unbounded space characterized by the lower phase velocity of the denser surrounding medium. In turn, the equation is solved numerically via the method of moments (MoM). However, application of an explicit modeling scheme and solving the resultant lower triangular matrix by a standard marching-on-in-time (MOT) method cannot, in this case, lead to the correct solution [3], [4]. This, as was noted in [5], is due to the fact that, in the explicit scheme, any spatial interval is larger than the distance traveled by the waves during the specified time interval. Consequently, propagation of high-phase-velocity waves cannot be provided for.

In this paper, we present a solution based on the implicit modeling scheme [6]–[9], which overcomes the deficiency of the MOT procedure, as well as the complexity involved in solving the implicit MoM equation. The solution is obtained via a novel method, which employs a spatio–temporal wavelet basis to facilitate an accurate solution simultaneously at all time steps within the time frame of interest. The wavelet-based MoM solution of the time-domain integral equation (TDIE) is effected via the iterative impedance-matrix-compression (IMC) procedure, which gradually constructs and solves a compressed version of the matrix equation until the desired level of accuracy is obtained [10]–[12].

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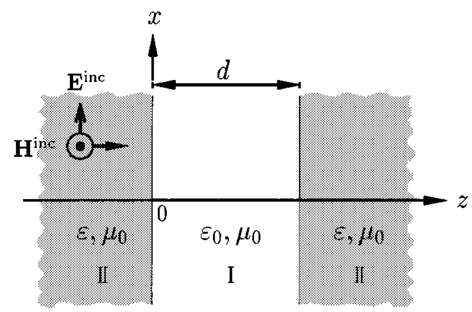

Fig. 1. Scattering of electromagnetic plane wave by an air layer (I) in a dielectric medium (II).

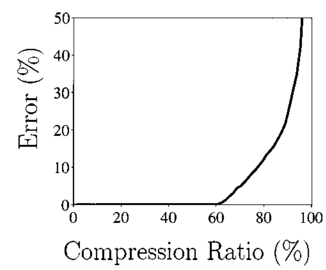

Fig. 2. Error versus compression ratio.

This paper is organized as follows. Section II presents the problem and outlines the method of solution. Numerical results are given in Section III. Finally, a summary and conclusions are given in Section IV.

II. Problem Formulation

The problem under consideration is the one-dimensional transient interaction of a Gaussian pulse of an electromagnetic plane wave with an air layer in a dielectric medium, as depicted in Fig. 1.

The medium has a dielectric constant $\varepsilon > \varepsilon_0$ and the air layer fills the entire space between planes $z = 0$ and $z = d$. The incident Gaussian pulse, propagating along the z-direction with its electric-field vector parallel to the surface of the layer, is given by

$$\mathbf{E}^{\mathrm{inc}}(z,\, t) = E_x^{\mathrm{inc}}(z,\, t)\hat{\mathbf{x}} = E_0^{\mathrm{inc}} e^{-(\tau/\Delta T)^2} \cos(\omega_0 \tau)\hat{\mathbf{x}}. \quad (1)$$

Here, ω_0 and ΔT denote, respectively, the pulse central frequency and width. Also, $\tau = t - t_{\mathrm{d}} - z/c$, where t_{d} denotes the time at which the peak of the pulse impinges on the $z = 0$ plane, and $c = 1/\sqrt{\varepsilon\mu_0}$ is the speed of light in the surrounding dielectric medium. We are interested in finding the yet unknown total electric field E_x expressed as

$$E_x(z,\, t) = \begin{cases} E_x^{\mathrm{I}}(z,\, t), & 0 \le z \le d \\ E_x^{\mathrm{II}}(z,\, t), & z < 0,\, z > d \end{cases} \quad (2)$$

where E_x^{I} and E_x^{II} are the total electric fields in the air layer and the surrounding dielectric medium, respectively. The integral equation for the problem under consideration is obtained by first introducing an equivalent polarization current, radiating in free space, defined as

$$J_x^e(z,\, t) = (\varepsilon - \varepsilon_0)\, \frac{\partial}{\partial t}\, E_x^{\mathrm{I}}(z,\, t), \qquad 0 \le z \le d. \quad (3)$$

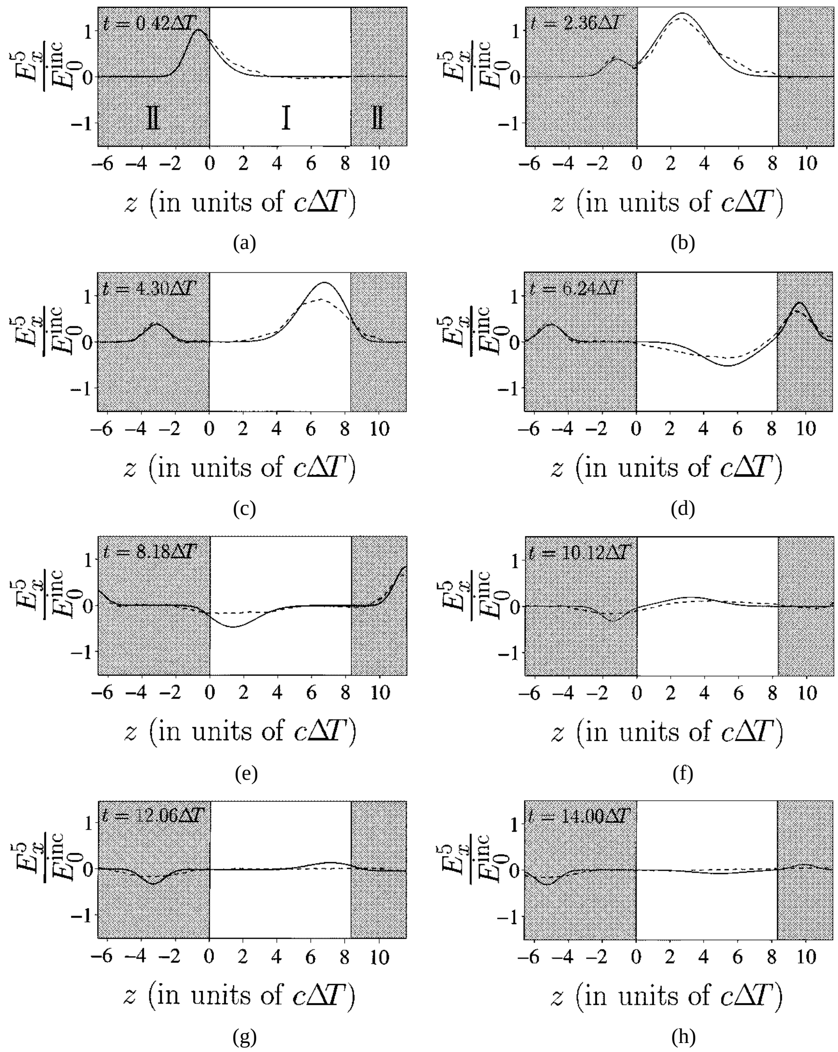

Fig. 3. Normalized electric field at various time steps. The dashed line denotes the solution obtained via the iterative IMC procedure after five iterations (640 basis functions), while the solid line presents the analytic solution.

Using the appropriate Green's function

$$g(z,\, t) = -\frac{\mu_0 c}{2}\,\delta\left(t - \frac{|z|}{c}\right) \tag{4}$$

the convolution of (3) with (4), in space as well as time, together with the appropriate continuity condition, yields the TDIE for the problem [3], [4]. We have

$$E_x^{\mathrm{I}}(z,\, t) + \frac{\mu_0 c(\varepsilon_0 - \varepsilon)}{2}\int_0^d \mathrm{d}z'\,\frac{\partial}{\partial\tau}\,E_x^{\mathrm{I}}(z',\, \tau)\bigg|_{\tau = t - (|z - z'|/c)} = E_x^{\mathrm{inc}}(z,\, t), \qquad 0 \le z \le d. \tag{5}$$

Then, by defining the operator

$$\mathcal{L}(\bullet) = (\bullet) + \frac{\mu_0 c(\varepsilon_0 - \varepsilon)}{2}\int_0^d \mathrm{d}z'\,\frac{\partial}{\partial\tau}(\bullet)\bigg|_{\tau = t - (|z - z'|/c)}, \quad 0 \le z \le d \tag{6}$$

one obtains

$$\mathcal{L}\left(E_x^{\mathrm{I}}(z,\, t)\right) = E_x^{\mathrm{inc}}(z,\, t), \qquad 0 \le z \le d. \tag{7}$$

To cast (7) into a matrix form, we apply the MoM [13] and span the unknown field E_x^{I} in terms of two sets of basis functions $\{\mathcal{U}_n(z)\}_{n=1}^{N_{\mathrm{S}}}$ and $\{\mathcal{T}_p(t)\}_{p=1}^{N_{\mathrm{T}}}$ as follows:

$$E_x^{\mathrm{I}}(z,\, t) = \sum_{n=1}^{N_{\mathrm{S}}}\sum_{p=1}^{N_{\mathrm{T}}} a_{np}\,\mathcal{U}_n(z)\,\mathcal{T}_p(t), \qquad 0 \le z \le d \tag{8}$$

where the a_{np} are unknown coefficients. For simplicity, and without loss of generality, it is assumed that these functions are standard pulse basis functions. Substituting (7) into (8), and applying a Galerkin's method, one then arrives at

$$Z_{(N_{\mathrm{T}}\times N_{\mathrm{S}})\times(N_{\mathrm{T}}\times N_{\mathrm{S}})}\vec{I}_{(N_{\mathrm{T}}\times N_{\mathrm{S}})\times 1} = \vec{V}_{(N_{\mathrm{T}}\times N_{\mathrm{S}})\times 1} \tag{9}$$

where

$$Z_{ij} = \left\langle \mathcal{U}_m(z),\, \left\langle \mathcal{T}_q(t),\, \mathcal{L}\Big(\mathcal{U}_n(z)\mathcal{T}_p(t)\Big)\right\rangle\right\rangle, \quad i = m + (q-1)\times N_{\mathrm{S}}; \quad j = n + (p-1)\times N_{\mathrm{S}} \tag{10}$$

$$I_j = a_{np}, \qquad j = n + (p-1)\times N_{\mathrm{S}} \tag{11}$$

$$V_i = \left\langle \mathcal{U}_m(z),\, \left\langle \mathcal{T}_q(t),\, E_x^{\mathrm{inc}}(z,\, t)\right\rangle\right\rangle, \qquad i = m + (q-1)\times N_{\mathrm{S}} \tag{12}$$

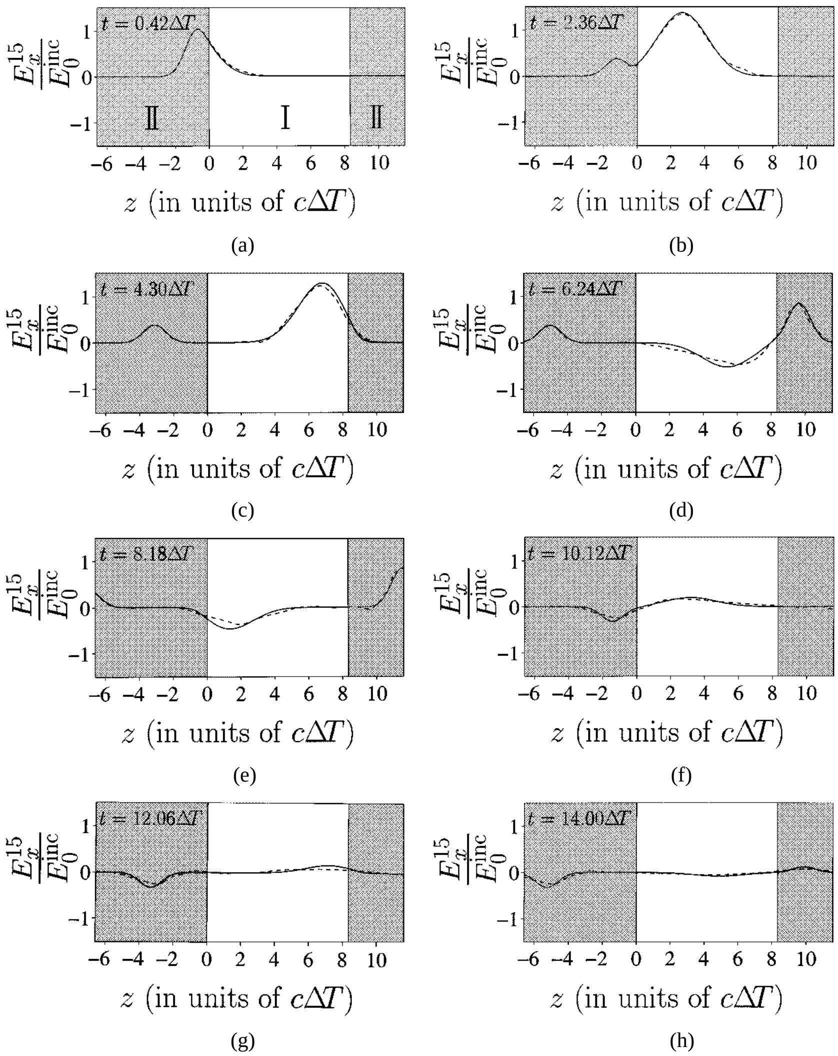

Fig. 4. Normalized electric field at various time steps. The dashed line denotes the solution obtained via the iterative IMC procedure after 15 iterations (1920 basis functions), while the solid line presents the analytic solution.

and

$$\langle f, g\rangle \equiv \int_{-\infty}^{\infty} f(\xi)g(\xi)\,\mathrm{d}\xi. \tag{13}$$

A solution of (9) for $\vec{I}$ determines the unknown coefficients a_{np}, which yield the electric field in the air layer E_x^{I} via (8). Using E_x^{I}, the electric field in the surrounding medium E_x^{II} can then be readily derived from

$$E_x^{\mathrm{II}}(z, t) = E_x^{\mathrm{inc}}(z, t) - \frac{\mu_0 c(\varepsilon_0 - \varepsilon)}{2}\int_0^d \mathrm{d}z' \frac{\partial}{\partial \tau} \cdot E_x^{\mathrm{I}}(z', \tau)\Big|_{\tau = t-(|z-z'|/c)}, \qquad z < 0, \quad z > d. \tag{14}$$

The discretization scheme for the combined spatial and temporal domains is either explicit or implicit. In the explicit scheme, the spatial and temporal sampling intervals Δz and Δt, which are chosen to meet the resolution requirements in their respective domains, obey

$$\Delta z \geq c\Delta t \tag{15}$$

while in the implicit scheme, they obey

$$\Delta z < c\Delta t. \tag{16}$$

The explicit scheme requires that the centers of the spatial pulse basis functions be at least as far apart as the distance that the wave propagating at velocity c can travel within each time step, thus, there is no interaction between any two such functions within the same time interval. The resultant matrix equation in this case is lower triangular and its solution can be obtained by MOT. However, the fact that the waves emerging from one basis function do not reach the adjacent functions within a given time step also implies that the explicit scheme cannot provide for the propagation of high phase-velocity (c_0) waves in the air layer [5], and the solution obtained in this way will never be the correct one. On the other hand, (16) ensures that each basis function interacts at least with its adjacent neighbors, thus, there exists a virtual path of propagation between any two basis functions within any given time step. This means that the effective propagation velocity can be higher than c, and reach the desired value of c_0. It may also be added that the implicit scheme is the more logical one to employ considering bandwidth and sampling requirements. Specifically, the value of Δt is set first according to the *a priori* known temporal bandwidth of the incident pulse. In turn, Δz is set to meet the smallest value required to match the finest spatial desired features of the unknown.

Nevertheless, the implicit scheme *does* require solving matrix equations. Obviously, solving (9) for all locations, and simultaneously for all the times, requires using almost all the original pulse basis functions, which brings about the need for an efficient method of solution. In

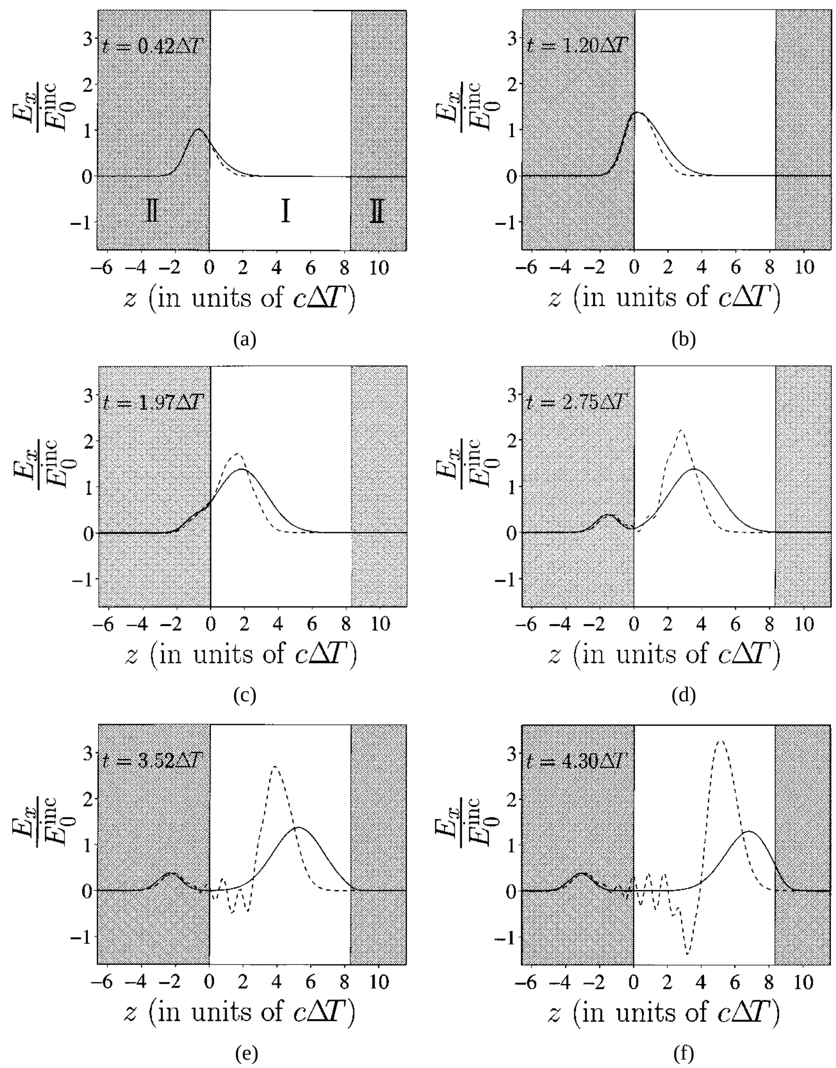

Fig. 5. Normalized electric field at various time steps. Each figure illustrates the analytic solution (solid line) and the one obtained using the explicit scheme (dashed line).

this study, we consider transforming (9) into a spatio–temporal wavelet basis [14], [15], and exploiting the sparsity of the solution in this representation (for simplicity, we have chosen to use the Haar wavelet basis, which is easy to implement. A more elaborate discussion of the use of spatio–temporal basis functions and the Haar wavelet functions can be found in [10]–[12]). To reduce the number of required basis functions, while keeping the desired level of accuracy, we use an alternative solution technique that has been proposed in [12]. Starting with an initial guess of the solution, the spatio–temporal Haar wavelet analysis is applied to extract the dominant expansion functions. Based on these dominant functions, a reduced (compressed) matrix equation is then constructed and solved. To refine the solution, the compression continues iteratively, adding on more expansion functions until the desired level of accuracy is achieved. The solution of the matrix equation at each stage is effected by a conjugate-gradient (CG) procedure. The CG procedure is rendered fast, converging upon taking as an initial guess the solution arrived at in the preceding stage.

III. Numerical Results

Numerical results are given here for the solution of the problem described in Section II. Considering the given computation resources, the various parameters have been set as follows: $\Delta T = 0.224$ ns, $t_{\rm d} = \Delta T$, $f_0 = \omega_0/2\pi = 0.375$ GHz, $d = 0.25\ {\rm m} = 8.33c\Delta T$, $\varepsilon = 5\varepsilon_0$, $N_{\rm S} = 64$, $N_{\rm T} = 128$, i.e., 8192 unknowns. In this solution, spatio–temporal Haar wavelet basis functions were used. Also, 128 functions were selected at each stage of the iterative IMC procedure. To study the performance of the new method, we first define the error in the solution at the lth iteration as

$$\text{Error} = \frac{\|E_x^l - E_x^{\rm REF}\|_2}{\|E_x^{\rm REF}\|_2} \tag{17}$$

where E_x^l is the solution at the lth iteration, $E_x^{\rm REF}$ is the analytic solution of this problem obtained via time–harmonic analysis followed by inverse Fourier transform, and

$$\|E(z,t)\|_2 = \frac{1}{\sqrt{d \times T_{\rm end}}}\sqrt{\int_0^d {\rm d}z \int_0^{T_{\rm end}} {\rm d}t |E(z,t)|^2} \tag{18}$$

where $T_{\rm end}$ is the temporal duration of interest. In our example, $T_{\rm end} = 5.5$ ns. This error is plotted in Fig. 2 as a function of the compression ratio, which is defined as

$$\text{Compression Ratio} = 1 - \frac{(\tilde{N}_{\rm T} \times \tilde{N}_{\rm S})^2}{(N_{\rm T} \times N_{\rm S})^2} \tag{19}$$

where $(\tilde{N}_{\rm T} \times \tilde{N}_{\rm S})$ is the dimension of the square compressed matrix. The compression ratio is the figure-of-merit for the size of the matrix equation actually solved (compressed matrix) in comparison to the original full-size matrix. Figs. 3 and 4 show the electric field at various time steps after the fifth (640 out of 8192 basis functions and compression of 92%) and fifteenth iterations (1920 out of 8192 basis functions and compression of 77%), respectively, together with the reference result $E_x^{\rm REF}$.

Finally, for comparative purposes, Fig. 5 illustrates a few results obtained using the explicit scheme (dashed line). Obviously, the explicit scheme does not yield the right solution (solid line). The results given here are for a shorter time duration than displayed in Figs. 3 and 4. This duration is limited because the explicit solution very soon becomes unstable. Also, the time intervals between successive graphs are shorter than in Figs. 3 and 4. This is done in order to illustrate in more detail the inherent lag of the explicit solution behind the right one.

IV. Summary and Conclusions

A wavelet-based MoM analysis of an implicit TDIE formulation for the problem of electromagnetic pulse interaction with an air layer in a dielectric medium has been presented. We have demonstrated the inadequacy of the explicit formulation in this case, and have showed that only the implicit formulation is the proper one for analyzing the problem at hand. In addition, the solution has been augmented by applying the IMC method. This is a stable iterative procedure, whereby the solution accuracy, at every spatial location and simultaneously at all the times of interest, is gradually refined.

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Investigation of Static and Quasi-Static Fields Inherent to the Pulsed FDTD Method

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***Abstract*—This paper demonstrates that trailing dc offsets, which can affect $\mathbf{E}$- or $\mathbf{H}$-fields in finite-difference time-domain simulations, are physically correct static solutions of Maxwell's equations instead of being numerically induced artifacts. It is shown that they are present on the grid when sources are used, which generates nondecaying charges. Static solutions are investigated by exciting electric and magnetic dipoles models with suitable waveforms.**

***Index Terms*—Divergence, FDTD method, Hertzian dipole, infinitesimal current element, triple cosine.**

I. Introduction

The pulsed finite-difference time-domain (FDTD) method [1] is based on a transitory system excitation coupled with the Fourier transform of its response. Excitation is introduced either by "soft" (added) or "hard" (fixed) sources [2]. We use soft sources to model elementary (or Hertzian) electric $\mathbf{p}$ and magnetic $\mathbf{m}$ time-varying dipoles, and compare FDTD results with the analytic solutions, in the static and quasi-static cases.

II. Static Fields in FDTD

The FDTD algorithm solves an initial boundary-value problem using only Maxwell's curl equations. Let us investigate briefly the way in which these vector equations determine the fields' temporal behavior. To this end, we need to study the divergence of $\mathbf{B}$ and $\mathbf{D}$, assuming sources defined as an impressed current density term $\mathbf{J}_s$ in the $\nabla \times \mathbf{H}$ equation. Taking the divergence of both sides of the curl equations, with the initial values $(\nabla \cdot \mathbf{B})_{t=0} \equiv 0$ and $(\nabla \cdot \mathbf{D})_{t=0} \equiv 0$, we notice the following.

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